Peer-Reviewed Open Access Journal ISSN 2455-1341 https://www.ijoret.com/

RESEARCH ARTICLE OPEN ACCESS

PULSATILE FLOW OF BLOOD WITH VOLUME FRACTION OF MICRO-ORGANISMS

BADERUNNISA BEGUM[1]

Assistant Professor, Department of Mathematics

1. Govt. First Grade College Shahapur, Dist:Yadgir, Karnataka, India

Email:badermath0506@gmail.com

Abstract

In this paper the pulsatile flow of blood with volume fraction of microorganisms has been planned to study by assuming blood as couple stress fluid and dusty particles as microorganisms. The velocity expressions for both blood and microorganisms have been obtained in Bessel-Fourier series form. By the proper substitution the velocity expressions for blood can be reduced to pulsatile and steady flow of couple-stress and Newtonian fluid in the presence/absence of microorganisms. Further from the present analysis the expressions for flow rate, apparent viscosity, phase-lag between flow rate and pressure gradient have been derived. The results obtained in this model will be compared with the existing experimental results and theoretical models.

Keywords: blood flow, dusty flow, dusty fluid flow.

1.Introduction

Blood is an important vehicle which carries oxygen from lungs onto tissues so also it transports nutrition to various tissues and so on so fourth. But blood is also a good media for the growth of microorganisms, thus leads to spread of (heamatogenic) infections which may be fatal or with high morbidity affecting the working hour of the man and inflicting on the economy of the nation.

A dusty fluid is a mixture of fluid and fine dust particles. Its study is important in areas like environmental pollution, smoke emission from vehicles, emission of effluents from industries, cooling effects of air conditioners, flying ash produced from thermal reactors and formation of rain drops etc. also it is useful in the study of lunar ash flow which explains many features of lunar soil.

The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field has important application in such areas as magneto hydrodynamics generators, pumps, accelerators, cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplets sprays. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators.

Hazem [1] has studied unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Gireesha et al., [2] have discussed the flow of unsteady dusty fluid under varying pulsatile pressure gradient in Anholonomic co-ordinate system. Saffman [3] has discussed the stability of the laminar flow the dusty gas in which the dust particles are uniformly distributed. Rao [4] have obtained the analytical solutions for the dusty fluid flow through a circular tube under the influence of constant pressure gradient, using appropriate boundary conditions. Later Nabil [5] studied the effect of couple stresses on pulsatile hydromagnetic poiseuille flow, and Dutta [6] obtained the solutions for pulsatile flow of heat transfer of a dusty fluid through an infinitely long annular pipe. Bagewadi et al., [7,8,9] have applied differential geometry techniques to investigate the kinematic properties of fluid flows in the field of fluid mechanics. Further B.J. Gireesha et al, [10,11] have studied two-dimensional dusty fluid flow in Frenet frame field system. Bugliarello & Sevilla [12] have studied the flow of unsteady dusty fluid under varying pressure gradients like constant, periodic and exponential.

The aim of the present investigation is to formulate and study the pulsatile flow of couple-stress fluid in the presence of dust particles. Further by assuming blood as couple stress fluid and dusty particles as microorganisms, analytical expressions are obtained for velocities of blood and microorganisms. The changes in the velocity profiles at different times are shown graphically.

2. FORMULATION OF THE PROBLEM:

The governing equations for unsteady, viscous, incompressible couple-stress fluid and microorganisms through a circular pipe are given by

$$(1-\phi)\rho\left\{\frac{\partial U}{\partial t}+(U.\nabla_1)U\right\}=(1-\phi)\left\{-gradP+\mu\nabla_1^2U-\eta\nabla_1^4U\right\}+KN_0(V-U) \quad (1)$$

$$\left\{\frac{\partial V}{\partial t}+(V.\nabla_1)V\right\}=\phi\left\{-gradP+\mu\nabla_1^2U-\eta\nabla_1^4U\right\}+\frac{K}{m}(U-V) \quad (2)$$

$$divU=0 \quad (3)$$

$$\frac{\partial N}{\partial t}+div(NV)=0 \quad (4)$$

Where U and V denote the local velocity vectors of fluid and microorganisms respectively, ρ - density, P- static fluid pressure, ν - kinematic viscosity, N_0 - number density of micro-organisms, K- Stoke's resistance coefficient ($6\pi\mu\varepsilon_1, \varepsilon_1$ - the radius of spherical particles), μ - the fluid viscosity, m – mass of microorganisms, ϕ - volume fraction and $\nabla_1^2=\left(\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\right)\left(r\frac{\partial}{\partial r}\right)$

For the present problem, the velocity distribution is taken only in Z – direction and is function of r and t only. Then the equations (1) and (2) reduces to

$$\left(1-\phi\right)\rho\frac{\partial w_1}{\partial t}=\left(1-\phi\right)\left[-\frac{\partial p}{\partial z}+\mu\nabla_1^2w_1-\eta\nabla_1^4w_1\right]+KN_0\left(w_2-w_1\right) \quad (5)$$

$$\frac{\partial w_2}{\partial t}=\phi\left[-\frac{\partial p}{\partial z}+\mu\nabla_1^2w_1-\eta\nabla_1^4w_1\right]+\frac{K}{m}(w_1-w_2) \quad (6)$$

Where w_1 and w_2 are the velocity components of blood and microorganisms respectively.

The boundary conditions are

w_1, w_2 and $\nabla_1^2(w_1, w_2)$ are all finite at r = 0

$w_1=0=w_2$ and $\nabla_1^2(w_1, w_2)=0$ at r = R (7)

Introduce the non-dimensional quantities,

$$u=\frac{w_1n\rho}{M},\ v=\frac{w_2n\rho}{M},\ y=\frac{r}{R},\ \phi^*=\frac{\phi}{b} \quad (8)$$

And assuming Womersley [16] type of pressure gradient of the form

$$-\frac{\partial p}{\partial z} = M\cos(nt+\theta) \tag{9}$$

Where M is the modulus, n is the circular frequency and θ is the phase in the complex representation of the Fourier coefficient of the velocity analogous to the pressure gradient gives,

$$\begin{aligned} u &= u_0(y)e^{\text{int}} \\ v &= v_0(y)e^{\text{int}} \end{aligned} \tag{10}$$

Using equations (4), (5) and (6), the equations (1) and (2) reduce to

$$(1-\phi)\left\{\nabla^4 u_0 - \bar{\alpha}^2\nabla^2 u_0 - \bar{\alpha}^2\alpha^2 e^{i\theta} + i\bar{\alpha}^2\alpha^2 u_0\right\} - \frac{\bar{\alpha}^2 b}{\beta}(v_0-u_0) = 0 \tag{11}$$

$$v_0 = \frac{-\phi\beta}{\bar{\alpha}^2 b(1+i\alpha^2\beta)}\left\{\nabla^4 u_0 - \bar{\alpha}^2\nabla^2 u_0 - \bar{\alpha}^2\alpha^2 e^{i\theta}\right\} + \frac{u_0}{1+i\alpha^2\beta} \tag{12}$$

Where $\bar{\alpha}^2 = \frac{R^2\mu}{\eta}$ is the couple – stress parameter, $\alpha^2 = \frac{R^2 n}{\nu}$ is the Pulsatile Reynolds' number, $b = \frac{mN}{\rho}$ is the mass concentration, $\beta = \frac{m\nu}{KR^2}$ is the relaxation time parameter, $i = (-1)^{1/2}$, $d = K/mn$ is Rathod – Gayatri (R-G) number, $\nu = \mu/\rho$ is kinematic viscosity.

The modified boundary conditions are

$$\begin{aligned} &u_0, v_0 \text{ and } \nabla^2(u_0, v_0) \text{ are all finite} \quad &\text{at } y=0 \\ &u_0 = 0 = v_0 \text{ and } \nabla^2(u_0, v_0) = 0 \quad &\text{at } y=1 \end{aligned} \tag{13}$$

$$\text{Where } \frac{1}{R^2}\nabla^2 = \frac{1}{R^2}\left[\frac{1}{y}\frac{d}{dy}\left[y\frac{d}{dy}\right]\right] = \nabla_1^2 \tag{14}$$

3. Solution of problem:

In order to solve the fourth order boundary value of blood flow problem, it is convenient to apply the finite Hankel transform with respect to y defined by (Tranter [16]).

$$\overline{u_0}(k) = \int_0^1 u_0(y) y J_0(ky) dy \tag{15}$$

Where k's are the positive roots of the equation

$$J_0(k) = 0 \tag{16}$$

$J_0(k)$ Denotes the Bessel function of the first kind of order zero

The Inverse transform is given by

$$\overline{u_0}(y) = 2\sum_k \overline{u_0}(k) \, {J_0(ky)} \Big/ {J_1^2(k)} \tag{17}$$

The summation is taken over all the positive roots of (16). Applying the Hankel transfrom (15) and inverse transform (17) to the equation (11) and by using (10) and boundary conditions (13), we get the velocity of blood in non-dimensional form as,

$$u(y) = 2\sum_k \left\{ \frac{\overline{\alpha}^2 \alpha^2 \cos\left(nt+\theta\right)}{\left[k(k^4 + \overline{\alpha}^2 k^2 + X)\right]} \right\} {J_0(ky)} \Big/ {J_1(k)} \tag{18}$$

Where $\quad X = \dfrac{\overline{\alpha}^2 \alpha^4 \beta\left(1-\phi b\right) b\left(1-\phi\right)}{1+\alpha^4\beta^2\left(1-\phi b\right)^2} \qquad (19)$

Using the equation (18) and (10) in (12), we get the velocity for microorganisms in non-dimensional form as

$$v(y) = 2\sum_k \left[\frac{\overline{\alpha}^2 \alpha^2 e^{i(nt+\theta)}}{\left[k(k^4 + \overline{\alpha}^2 k^2 + X)\right]} \left(M^* X \cos(nt+\theta) + \frac{1}{1+i\alpha^2\beta} \right) \right] {J_0(ky)} \Big/ {J_1(k)} \tag{20}$$

Where $M^* = \dfrac{\phi\beta}{\overline{\alpha}^2 (1+\alpha^4\beta)}$

Volumetric flow rate Q is defined by

$$Q = 2\pi \int_0^1 y\left(u(y), v(y)\right) dy \tag{21}$$

Using equations (18) and (20) in (21), we get

$$Q_u = 2\pi\left[2\sum_k\left[(\bar{\alpha}^2\alpha^2\cos(nt+\theta))\Big/\left\{k\left(k^4+\alpha^2k^2+X\right)\right\}\right]\right] \tag{22}$$

$$Q_v = 2\pi\left[2\sum_k\frac{1}{k}\left[\frac{\bar{\alpha}^2\alpha^2e^{i(nt+\theta)}}{k(k^{4+}\bar{\alpha}^2k^2+X)}\left(M^*X\cos(nt+\theta)+ 1\Big/1+\alpha^4\beta\right)\right]\right] \tag{23}$$

Where Q_u and Q_v are the flow rates of couple-stress fluid and microorganisms.

Apparent viscosity for Poiseuille flow is given by,

$$\mu_a = \frac{\pi R^4 M e^{i(nt+\theta)}}{8Q} \tag{24}$$

For the present problem the apparent viscosity for blood is,

$$\mu_b = \frac{\mu e^{i\theta}}{32(\bar{\alpha}^2\alpha^2e^{i\theta})}\sum_k k^2(k^4+\bar{\alpha}^2k^2+X) \tag{25}$$

Apparent viscosity for microorganisms is,

$$\mu_a = \frac{\mu e^{i\theta}}{32\sum_k\left[\frac{\bar{\alpha}^2\alpha^2e^{i\theta}}{k(k^4+\bar{\alpha}^2k^2+X)}\left(M^*Xe^{i\theta}+\frac{1}{1+\alpha^4\beta}\right)\right]} \tag{26}$$

4. RESULTS AND DISCUSSIONS

It is of interest to note that the expression for velocity in the present analysis (equation (18)) contains the expression for the velocity of Pulsatile and steady flow of (i) couple-stress fluid, (ii) Newtonian fluid, in the presence and absence of microorganisms. By setting m = 0 in the equation (19), the equation (18) will reduce to the Pulsatile flow of couple-stress fluid in the absence of micro-organisms as,

$$u(y) = 2\left(\bar{\alpha}^2\alpha^2\cos(nt+\theta)\right)\sum_k\frac{1}{\left(k\left(k^4+\bar{\alpha}^2k^2\right)\right)}\frac{J_0(ky)}{J_1(k)} \tag{27}$$

Velocity expression for steady couple-stress fluid without microorganisms is obtained by setting n = 0 and m = 0 in equation (18) and its expression in dimensional form is given by,

$$w_1 = \frac{2MR^2 \bar{\alpha}^2 e^{i\theta}}{\mu} \sum_k \frac{1}{k(k^4 + \bar{\alpha}^2 k^2)} \frac{J_0(ky)}{J_1(k)} \tag{28}$$

The velocity for pulsatile Newtonian fluid with microorganisms can be obtained by putting $\bar{\alpha} \to \infty$ in equation (18) and is given by

$$u(y) = 2e^{\text{int}}(\alpha^2 e^{i\theta}) \sum_k \frac{1}{k(k^2 + X)} \frac{J_0(ky)}{J_1(k)} \tag{29}$$

Where X= $\dfrac{\bar{\alpha}^2 \alpha^4 \beta(1-\phi b) b(1-\phi)}{1+\alpha^4 \beta^2 (1-\phi b)^2}$

By substituting n = 0 in expression (29) makes $d \to \infty$. Hence we get the velocity expression for steady Newtonian fluid without microorganism in dimensional form as,

$$w_1 = \frac{2R^2 M e^{i\theta}}{\mu} \sum_k \frac{1}{k^3} \frac{J_0(ky)}{J_1(k)} \tag{30}$$

By introducing m = 0 in (29), we get velocity expression for pulsatile flow of Newtonian fluid in the absence of microorganisms as,

$$u(y) = 2e^{\text{int}}(\alpha^2 e^{i\theta}) \sum_k \frac{1}{k(k^2)} \frac{J_0(ky)}{J_1(k)} \tag{31}$$

By setting n = 0 in (31), we get velocity equation for steady Newtonian fluid in the absence of microorganisms in dimensional form as,

$$w_1 = \frac{2R^2 M \bar{\alpha}^2 e^{i\theta}}{\mu} \sum_k \frac{1}{k^3} \frac{J_0(ky)}{J_1(k)} \tag{32}$$

By substituting $\bar{\alpha} \to \infty$ in equation (20), we get an expression for velocity of microorganisms through pulsatile flow of Newtonian fluid as,

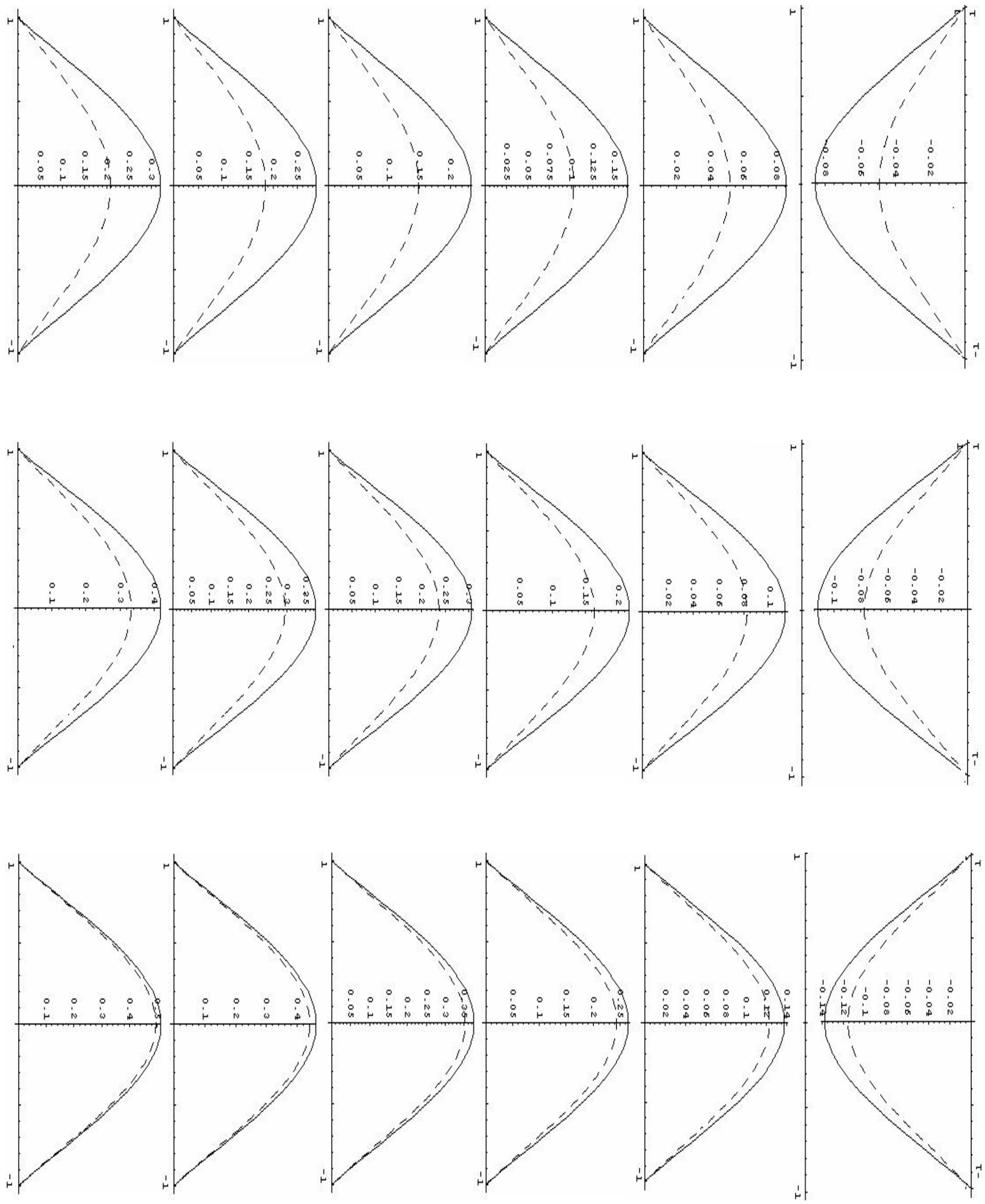
$$v(y) = 2\left\{ \frac{\alpha^2 e^{i(nt+\theta)}}{k(k^2 + X)} \left[M^* X + \frac{1}{1 + i\alpha^2 \beta} \right] \right\} \times \frac{J_0(ky)}{J_1(k)} \tag{33}$$

Where $M^* = \dfrac{\phi\beta}{\overline{\alpha}^2 (1+\alpha^4\beta)}$, X= $\dfrac{\overline{\alpha}^2 \alpha^4 \beta (1-\phi b) b (1-\phi)}{1+\alpha^4\beta^2 (1-\phi b)^2}$

The expression for steady flow of Newtonian fluid (32) is obtained from the present model is exactly same. The velocity profiles obtained from the expression (32), (R = 20 μm, $\mu = 4$ cp, pressure gradient =67.5 dynes / mm^3).

The velocity profile for a steady couple-stress fluid have been computed using equation (8) (Milton and Stegun [15]) and are found to be in good agreement with the theoretical results of Ref [14] and experimental results of Bugliarello et al [12,13]. The velocity profiles obtained from the present model for a steady couple-stress fluid flow (equation (28) are exactly same as that of experimental results.

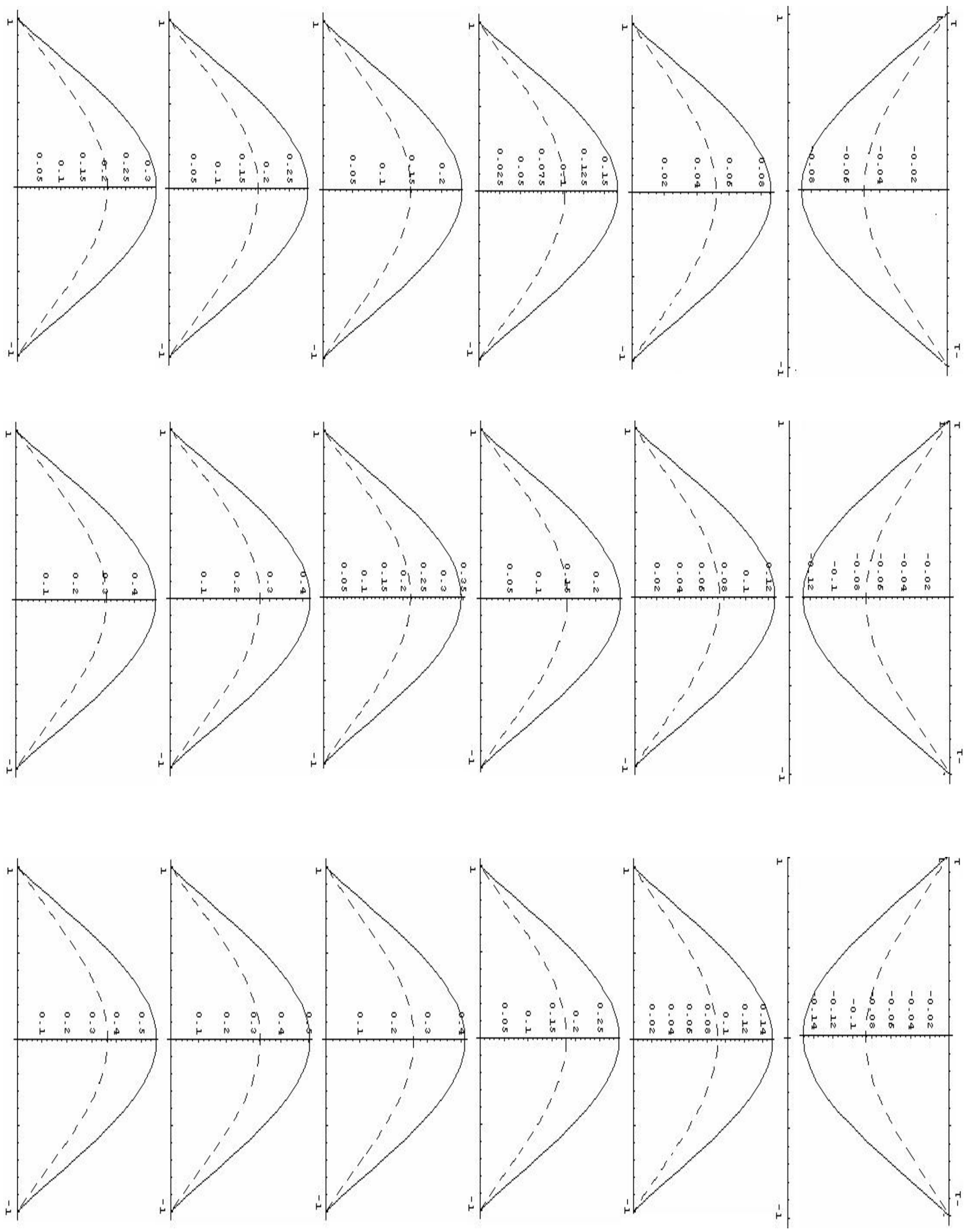
The velocity profile for three values of α ($\alpha^2 = 3, 4, 5$) and for different values of $\overline{\alpha}$ and nt (b=1, $\beta = 0.2$, $\phi = 0.5$, $\theta = 15^0$) have been shown through figure 1 and 2 for blood and microorganisms. From these figures it is clear that the velocity profile is parabolic upto nt = 60^0, at nt=75^0 the velocity is almost zero. Further as nt values increases the reverse flow for the blood has been observed. As the couple-stress parameter increases, the velocity also increases. It is of interest to note that when the pulsatile Reynolds number is increased, accordingly the velocity is also increased. As the values of $\overline{\alpha}$ and α increases, the corresponding increase in the velocity profile has been observed. It has been found that as the volume fraction is increased there is increase in the velocity for different values of pulsatile Reynolds number, & nt and in above both cases velocity is parabolic upto nt = 60^0 and at nt = 75^0 velocity is almost zero , further as nt value increases the reverse flow for both blood and micro-organisms has been observed.



nt=0^0 15^0 30^0 45^0 60^0 75^0 90^0

Fig.1.1 VELOCITY VARIATION WITH nt, α^2 =3,4,5

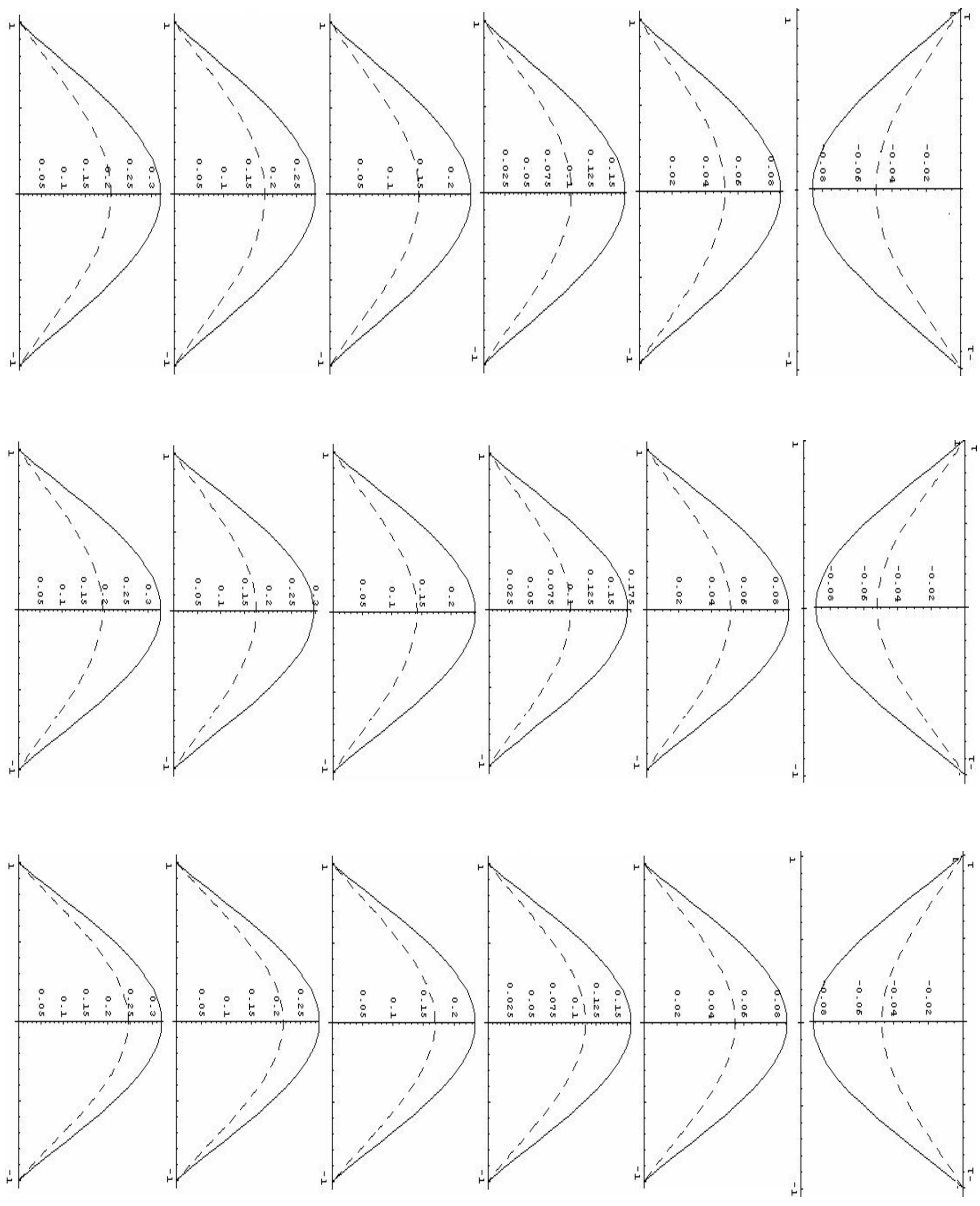
(b=1, β =0.2, θ =15° , $\overline{\alpha}$ = 2, ϕ =0.5)



nt=0⁰ 15⁰ 30⁰ 45⁰ 60⁰ 75⁰ 90⁰

Fig 1.2 VELOCITY VARIATION WITH nt, $\bar{\alpha}$ =2, 3, 4

(b=1, β=0.2**,** θ=15°, $\alpha^2 = 3$, ϕ=0.5**)**



nt=0^0 15^0 30^0 45^0 60^0 75^0 90^0

Fig 1.3 VELOCITY VARIATION WITH nt, ϕ = 0.5, 1.0, 1.5

(b=1, β=0.2, θ=15°, α^2= 3, $\overline{\alpha} = 2$ **)**

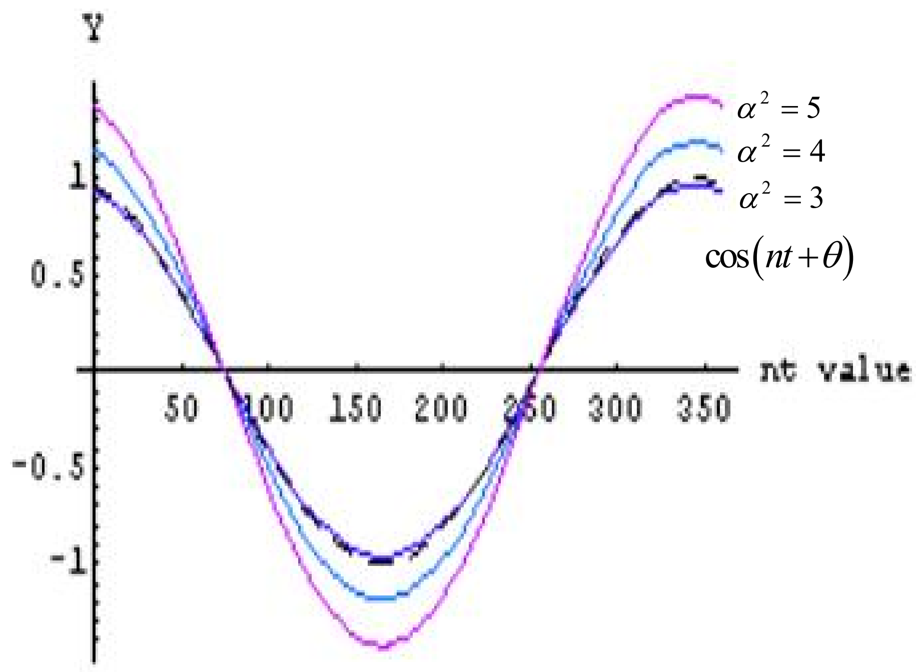


Fig 1.4. Variation of flow rate with nt for blood
(b=1, β=0.2, θ=15°, $\overline{\alpha} = 2$, ϕ=0.5)

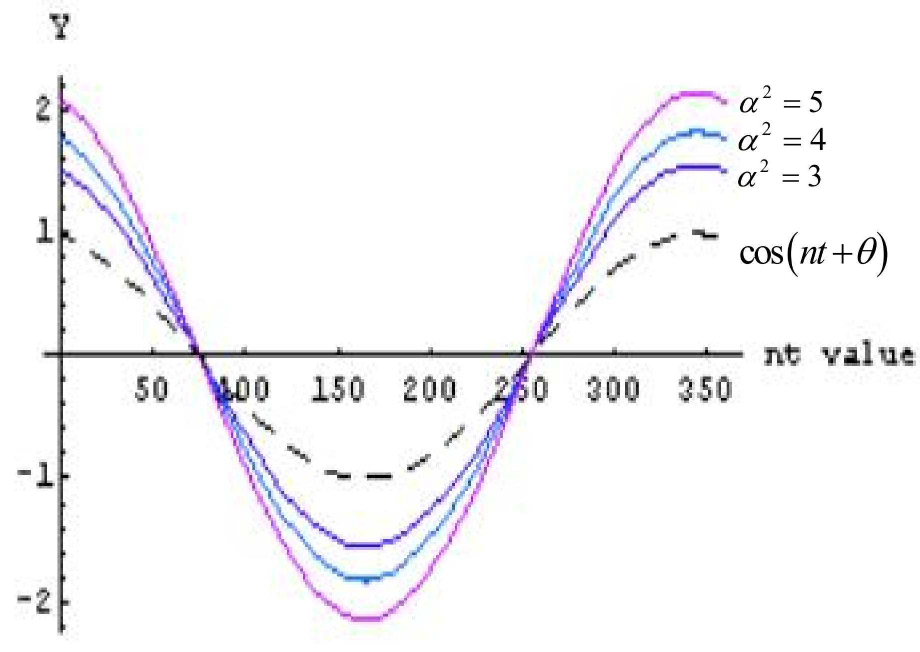


Fig 1.5. Variation of flow rate with nt for blood
(b=1, β=0.2, θ=15°, $\overline{\alpha} = 4$, ϕ=0.5)

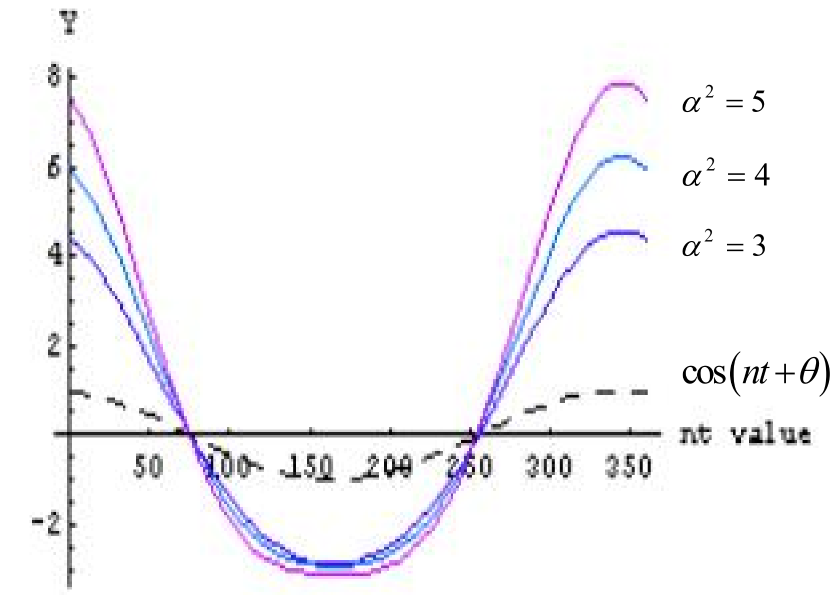


Fig 1.6. Variation of flow rate with nt for microorganisms
(b=1, β=0.2, θ=15°, $\overline{\alpha} = 2$, ϕ=0.5)

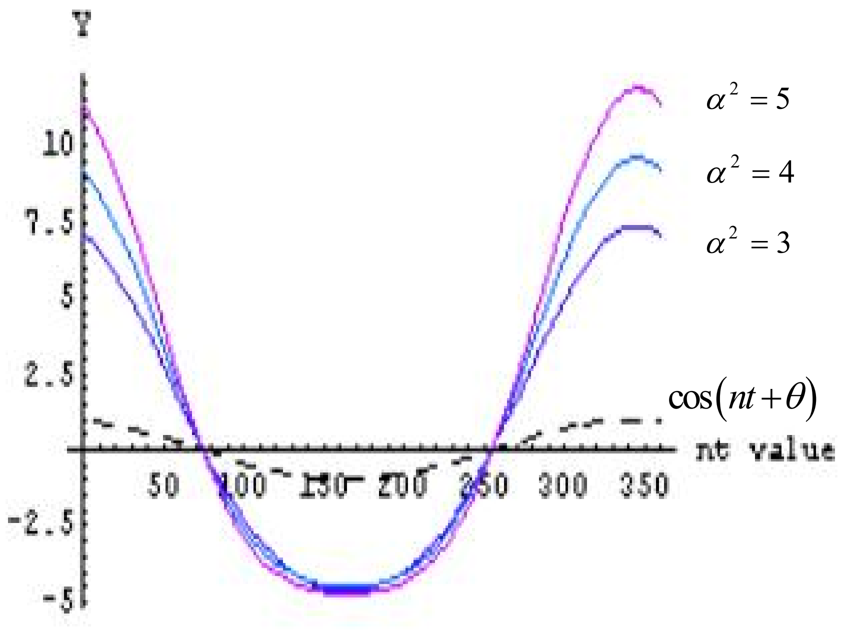


Fig 1.7. Variation of flow rate with nt for microorganisms
(b=1, β=0.2, θ=15°, $\overline{\alpha} = 4$, ϕ=0.5)

5. CONCLUSION:

The pulsatile flow of blood and microorganisms through rigid circular tube has been studied. The expressions for velocity, flow-rate and apparent viscosity for blood and microorganisms have been obtained in the form of Bessel-Fourier series. For different values of couple-stress parameter $\overline{\alpha}$, pulsatile Reynolds number α and pressure gradient the velocity profile have been obtained and shown through fig 1 & 2. It is observed that as $\overline{\alpha}$ increases, the axial velocity will also increase. The expression obtained for the velocity of blood in the present model in the presence of microorganisms can be reduced to the expression for velocity of blood only(in the absence of microorganisms) by substituting m = 0 in equation (28). When n = 0, this expression leads to steady couple-stress fluid, the velocity profiles obtained by this expression, for different values of pressure gradient and $\overline{\alpha}$ have been compared with the experimental results and they are found to be in good agreement with these experimental results of Bugliarello & Sevilla [12]. Hence, the results obtained from the present model converges fast. Further when $\overline{\alpha} = \infty$, we get an expression for the pulsatile Newtonian fluid. This expression will reduce to steady Newtonain fluid, when n = 0. This expression of steady Newtonian fluid, equation (28), is in better agreement with that of experimental results Bugliarello et al [13]. The flow rates have been computed for different values of α, $\overline{\alpha}$ and nt for 1^{st}, 2^{nd} and 3^{rd} approximations and a good convergence has been observed.

References:

1. Hazem A. ATTIA, Turk J. Phys, **29**, 257-267 (2005).
2. B.J. Gireesha, C.S. Bagewadi and B.C. Prasanna Kumara, EJTP 4, No.14,9-16 (2007)
3. P.G. Saffman, On the Stability of laminar flow of a dusty gas, Journal of Fluid Mechanics, 13**,**120-128 (1962).
4. P. Samba Siva Rao, Unsteady flow of a dusty viscous liquid through circular cylinder, Def. Sci. J., 19,135-138 (1969).
5. T.M. Nabil, EL- Dabe, M.G. Salwa and EL- Mohandis, Effect of couple stresses on Pulsatile hydromagnetic poiseuille flow, Fluid Dynamic Research, 15, 313-324 (1995).

6. N.Datta & D.C.Dalal, Pulsatile flow of heat transfer of a dusty fluid through an infinitely long annlur pipe, Int. J. Multiphase flow, **21(3),** 515-528 (1995).
7. C.S.Bagewadi and A.N.Shantharajappa, A study of unsteady dusty gas flow in Frenet Frame Field, Indian Journal pure Appl. Math. **31**, 1405-1420 (2000).
8. C.S.Bagewadi and B.J.Gireesha, A study of two dimensional steady dusty fluid flow under varying temperature, Int. Journal of Appl. Mech & Eng., **09,** 647-653 (2001).
9. C.S.Bagewadi and B.J.Gireesha, A study of two dimensional unsteady dusty fluid flow under varying pressure gradient, Tensor, N.S. 64, 232-240 (2003) .
10. B.J. Gireesha, C.S. Bagewadi & B.C.Prasannakumara, Flow of unsteady dusty fluid under varying periodic pressure gradient, Journal of Analysis and Computation, 2(2), 183-189 (2006).
11. B.J. Gireesha, C.S.Bagewadi & B.C.Prasannakumara, flow of unsteady dusty fluid between two parallel plates under constant pressure gradient, Tensor, N. S (2007).
12. Bugliarello.G & Sevilla, J, Velocity distribution and other characteristics of steady and Pulsatile blood flow in fine glass tubes, Biorheology, 7, 85-107 (1970).
13. Bugliarello.G., Kapur, C and Hsaio, G, The profile viscosity and other characteristics of blood flow in a non-uniform and shear field, jnt. Symposium on Biorheology, A.L. Compley (Ed). New York, jnterscience-john Wiley, 351-370 (1965).
14. Chaturani, P and Rathod, V.P. A critical study of poiseuille flow of couple stress fluid with applications to blood flow, Biorheology, 18, 235-244 (1981).
15. Milton, A and Stegun, J.A, Hand book of mathematical functions with formulas, Graphs and mathematical tables, Dover publications Inc., NewYork.
16. Tranter C.J., Editor: B.L. Wornshop, Integral transforms in mathematical physics, Infinite transforms, 8, 2nd edition Londan (1956).