Thermal characteristics of pseudoplastic non-Newtonian model with non-Fourier heat flux effect: A Numerical study

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Abstract

This study analyses the stagnation point flow of a non-Newtonian fluid past a stretching sheet, incorporating a non-Fourier heat flux model. By applying similarity transformations, the momentum, energy, and concentration equations are transformed into a system of ordinary differential equations. The numerical solution is obtained using the shooting method combined with the MATLAB software. The energy equation integrates the Cattaneo-Christov heat flux model. Velocity, temperature, and concentration profiles are examined for various fluid parameters, while key physical quantities such as the friction coefficient and Nusselt number are computed. Results indicate that an increase in the Williamson parameter enhances the Nusselt number while reducing the skin friction coefficient.

Keywords: Cattaneo-Christov heat flux, MHD, Williamson, Stretching sheet.

1. Introduction

In non-Newtonian fluids, the relation between shear stress and shear rate is always nonlinear which play key role in many applications such as polymer industries, food processing industries etc. Due to wide range application of boundary layer flow of non-Newtonian fluids many researchers investigated heat and mass transport phenomena in different types of non-Newtonian models. Initially Sakiadis(1–3) demonstrated boundary layer flows over moving surfaces with three different geometries. Later many researchers discussed boundary layer viscous flows problem with variety of physical conditions and geometries. Williamson fluid can be classified as visco-inelastic non-Newtonian model.Nadeem.S and Akram (4) studied about induced magnetic field significance in visco-inelstic fluid flows.Hayat.T et al (5) time dependent Williamson fluid flow over a surface with electric and magnetic field effects.In another study by Halim N.A et al (6) reports about heat and mass transport behaviour of Williamson fluid with zero mass flux boundary condition. Gorla R.S and Gireesha (7) given dual solution to stagnation point flow of visco elastic fluid flow over a moving surface.Konda Jayarama Reddy et al (8) examined non uniform heat source sink effect under melting heat transfer effect. Recently Tirumala et al (9) discussed impact of melting heat transfer in non-Newtonian fluids.Siginifcance of chemical reaction on a 3D stretching sheet problem was investigated by Hayat.T et al (10) using Williamson Rheological model. Recently Oguneseye S.A et al (11) discussed boundary layer behaviour of visco elastic model over a vertically moving surface.

Most of the cases one of the widely used expression for discussing heat transfer in various condition is Fourier Law. Later Cattaneo (12) modified this Fourier law with thermal relaxation time and later this model is further modified by Christov (13).Many researchers applied this model in analysing thermal properties of distinct non-Newtonian fluids in boundary layer flow problems. Fahad Munir Abbasi et al (14) considered this model for exploring Maxwell non Newtonian fluid flow over a stretching sheet.Anantha kumar et al (15) discussed impact variable heat source sink with different geometries using non-Fourier heat flux model.Sandeep and Sulocahna (16) developed model for discussing thermal behaviour of three different non-Newtonian fluids over a stretching surface with Cattaneo -Christov model. Recently Nazeer et al (17) scrutinised Carreau nanofluid flow over a stretching surface with variable fluid properties. Transpiration effect on fluid flow with Cattaneo-Christov model is presented by Raju C.S.K et al (18).Besides these above mentioned research works some more important works on this non-Fourier heat flux model can be found in the literature.(See (15,19–24)).Most of the authors cited above focused on Cattaneo-Christov model with different non-Newtonian models under different physical conditions ,but no work is reported on Visco-inelastic model to the best of authors knowledge. Hence main aim of this article is to explore thermal characteristics of Willianson stagnation point flow with Cattaneo-Christov model.

2. MATHEMATICAL MODEL

Let us consider a 2D incompressible, time independent laminar fluid past a stretching sheet. Flow is induced by stretching the surface in x-direction and a transverse magnetic field B_0 is taken perpendicular to surface.T_w and C_w indicates constant temperature and concentration at the sheet Free stream velocity is taken as $u_e = cx$ and stretching sheet velocity is $u_w = ax$ .The governing boundary layer equations for MHD convective Williamson fluid flow behaviour of Cattaneo-Christov heat flux type over a linear stretched surface can be written as

Continuity equation: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial x^2}\right)+\sqrt{2}\Gamma v\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)+u_e\frac{du_e}{dx}+-\frac{\sigma B_0^2}{\rho}(u-u_e) \quad (2)$$

Energy equation :

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$$
$$+\lambda\left[u^2\left(\frac{\partial^2 T}{\partial x^2}\right)+2uv\left(\frac{\partial^2 T}{\partial x\partial y}\right)+v^2\left(\frac{\partial^2 T}{\partial y^2}\right)+u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+u\frac{\partial T}{\partial x}\frac{\partial u}{\partial x}+v\frac{\partial T}{\partial x}\frac{\partial u}{\partial y}\right] \quad (3)$$

Concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

With boundary conditions,

$$u=U_w=cx, v=0, -k\frac{\partial T}{\partial y}=h_f\left(T_w-T\right), C=C_w \text{ at } \quad y=0$$

$$u\to u_e=ax, T\to T_\infty, C\to C_\infty \text{ at } \quad y\to\infty \quad (5)$$

Here velocity components in x and y direction are represented by u and v respectively and u_e refers to free stream velocity , strength of magnetic field is denoted by B_0 , Γ is the time rate constant .The governing equations (1)-(4) can be transformed to a set of nonlinear ordinary equations by introducing the following non-dimensional variables :

$$\eta=\sqrt{\frac{c}{\upsilon}}y,\ u=cxf'(\eta),\ v=-\sqrt{c\upsilon}\ f(\eta)\ \ \theta=\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\phi=\quad \emptyset(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (6)$$

with application of equation-(6) , fluid flow governing equation takes the following form.

$$f'''(\eta)+f(\eta)f''(\eta)-f'(\eta)^2+w_e f'''(\eta)f''(\eta)-M\left[f'(\eta)-\lambda\right]+\lambda^2=0 \quad (7)$$

$$\theta''(\eta)-P_r\gamma f(\eta)^2\theta''(\eta)-P_r\gamma ff'(\eta)\theta'(\eta)+P_r f(\eta)\theta'(\eta)+P_r Nb\theta\phi+Nt\theta^2=0 \quad (8)$$

$$\phi''(\eta) + S_c f(\eta)\phi'(\eta) + \frac{Nt}{Nb}\theta'' = 0 \quad (9)$$

The transformed boundary conditions:

$$\left.\begin{array}{l} f(0)=0, f'(0)=1, \theta'(0)=-\gamma_1\left[1-\theta(0)\right], \phi(0)=1 \\ at\eta=0 \\ f'(\infty)\rightarrow A, \theta(\infty)\rightarrow 0, \phi(\infty)\rightarrow 0 as\eta\rightarrow\infty \end{array}\right\}(10)$$

here M=$\frac{\sigma B_0^2}{c_\rho}$, is magnetic parameter , $W_e = U_w\sqrt{\frac{2c}{v}}\Gamma$ denotes Williamson fluid parameter, Boit number is denoted by $\gamma_1 = \frac{h_f}{k}\sqrt{\frac{v}{a}}$, and $S_C = \frac{v}{D_m}$ refers to Schmidt number

the local skin-friction $(C_{f,x})$ the local Nusselt number $(N_{u,x})$ and the local Sherwood number $(S_{h,x})$ are defined as $C_{f,x} = \frac{\tau_w}{\rho U_w^2}$; $N_{u,x} = \frac{xq_w}{c_p(T_w - T_\infty)}$; $S_{h,x} = \frac{xq_m}{D_m(C_w - C_\infty)}$ (11)

and these physical quantities can be reduced into the dimensionless form as below:

$$2\sqrt{2}C_{f,x}Re_x^{\frac{1}{2}} = f''(0) + W_e f''(0)^2 \qquad Sh_x Re_x^{\frac{1}{2}} = -\phi'(0), \qquad Nu_x Re_x^{\frac{1}{2}} = -\theta'(0) \quad (13)$$

Where $Re_x = \frac{xU_w}{v}$ is the Reynolds number.

3. Results and discussions.

In this section, we have discussed about behaviour of momentum, temperature and concentration distributions with respect to different fluid flow parameters such as Magnetic field parameter (M), Williamson parameter (We), stretching ratio parameter (A), Brownian motion (Nb), thermophoresis (Nt), Deborah number (λ) and Biot number (γ) etc. The governing equations (8)-(10) are solved along with boundary conditions mentioned equation (11) using shooting method. Fig 2 portrays the effect of magnetic parameter on momentum behaviour and it is noticed that velocity profiles decrease with strong magnetic field strength. The main reason for this is Lorentz force created due to applied transverse magnetic field in the perpendicular direction of fluid flow.

Fig 3 shows the temperature profile for magnetic field parameter (M) and clearly one can notice that temperature profile increases with respect to applied magnetic field. Impact of

Williamson parameter (We) is plotted through Fig 4 and from this picture it is clear that velocity distribution is decreased for higher values of parameter (We). Biot number effect on temperature profile is shown in Fig 5 and from this figure it is evident that temperature profile increases.

Fig 6 shows the significance Brownian motion parameter (Nb) on concentration profile and an decreased concertation profile noticed for higher Nb values.This happens due to switching over of fluid particles between cold region to hot region within the boundary layer. The impact of Thermophoresis on temperature is plotted through Fig 7 and an uplift of temperature distribution and thermal boundary layer thickness is noticed for thermophoresis parameter (Nt).

Fig 8 reveals the importance of Schmidt number (Sc) on concentration profile and it is clear that concentration profile decreases with Sc values. Basically, Schmidt number is ratio between molecular and viscous diffusion rate , so by increasing Sc values kinematic viscosity beats the mass diffusivity ,as a result concentration profile diminishes. Fig 9 exhibits role of stretching ratio parameter(A) on velocity profile and it is observed that velocity profile increases for A>1.

4. Conclusions

Williamson non-Newtonian fluid flow over a stretching sheet with Cattaneo-Christov heat flux is studied numerically with Buingrno model under convective heat transfer boundary condition using shooting method .Some of the important outcomes of this theoretical study are as follows:

- Stretching ratio parameter increases velocity profile.
- Skin friction coefficient decreases for Williamson parameter whereas Nusselt number increases for Williamson parameter.
- Sherwood number improved for Williamson parameter.
- Deborah number enhances temperature profile.

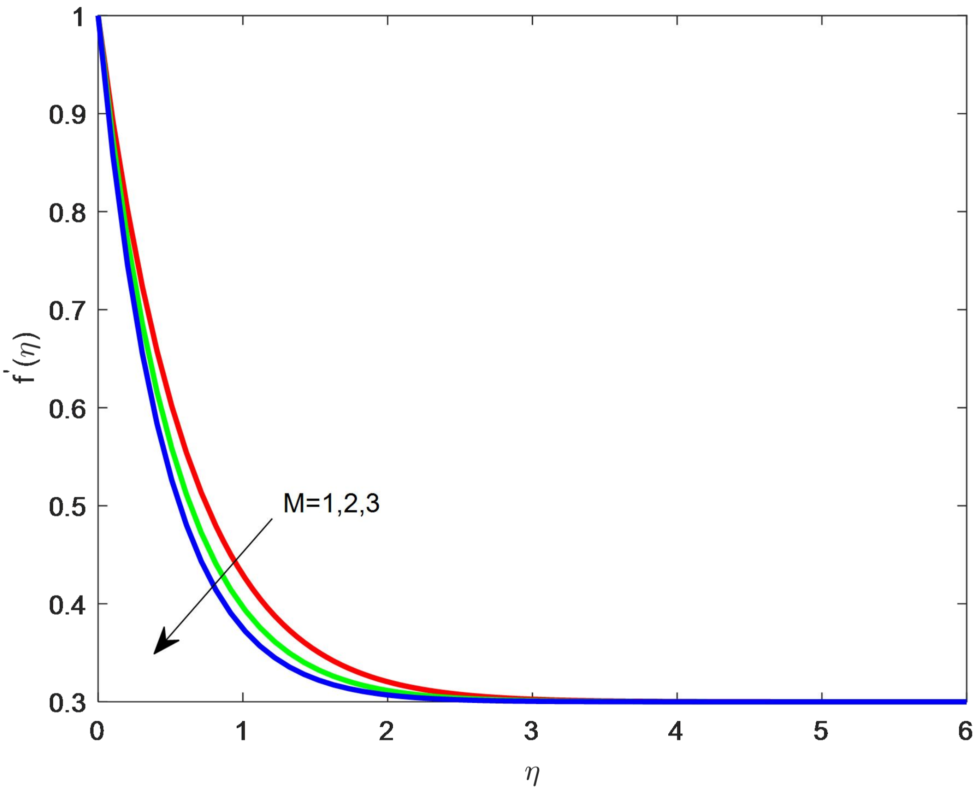

Fig 2: Impression of magnetic field on velocity.

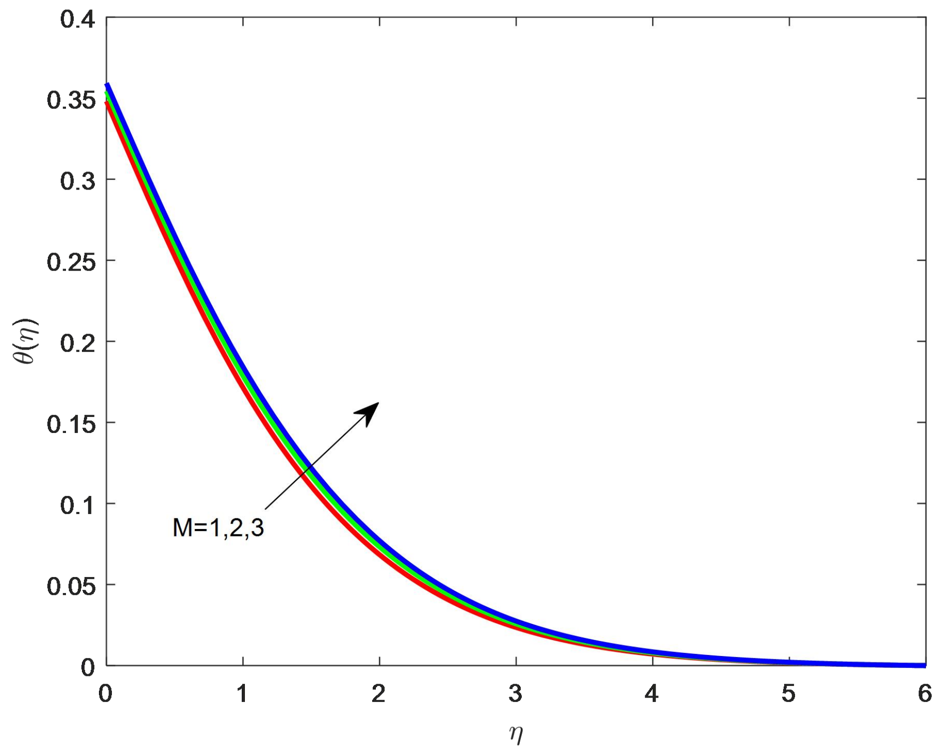

Fig 3: Impression of magnetic field on temperature.

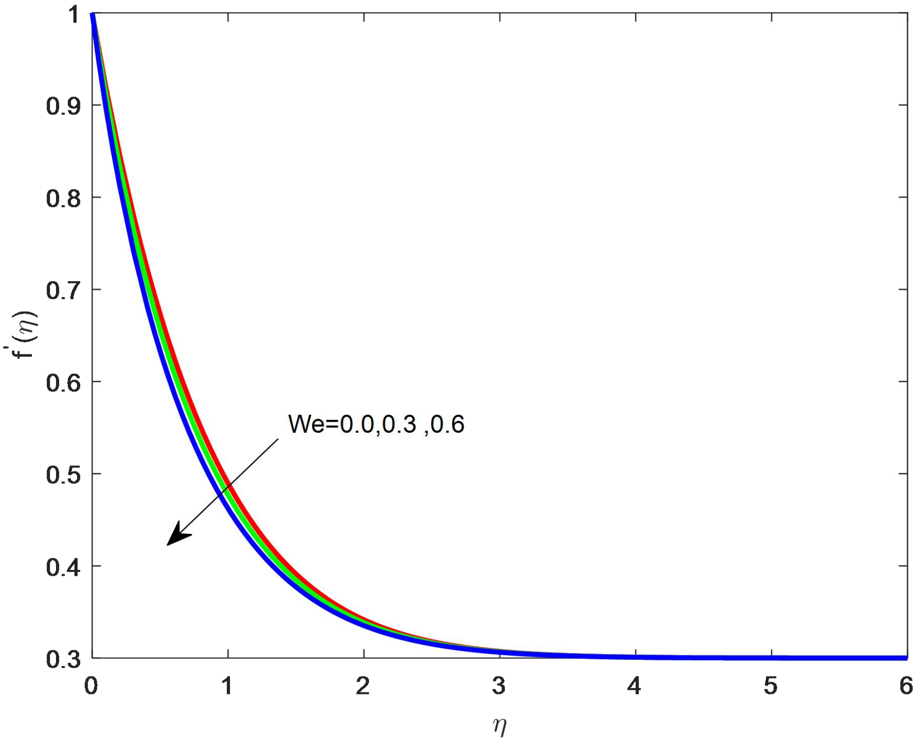

Fig 4 : Impression of Williamson parameter on velocity.

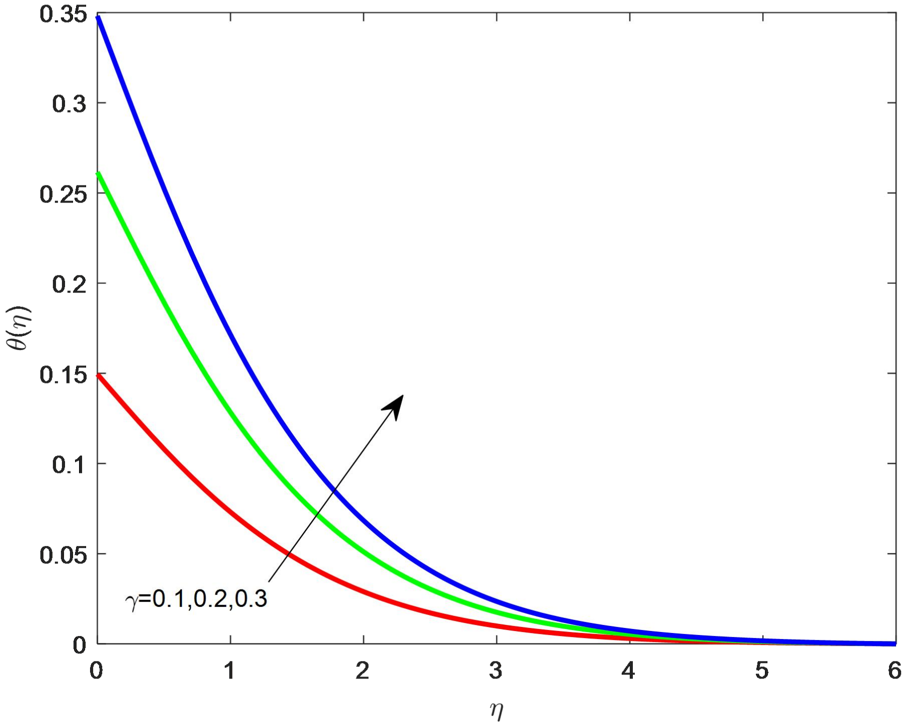

Fig 5 : Impression of Biot number on temperature.

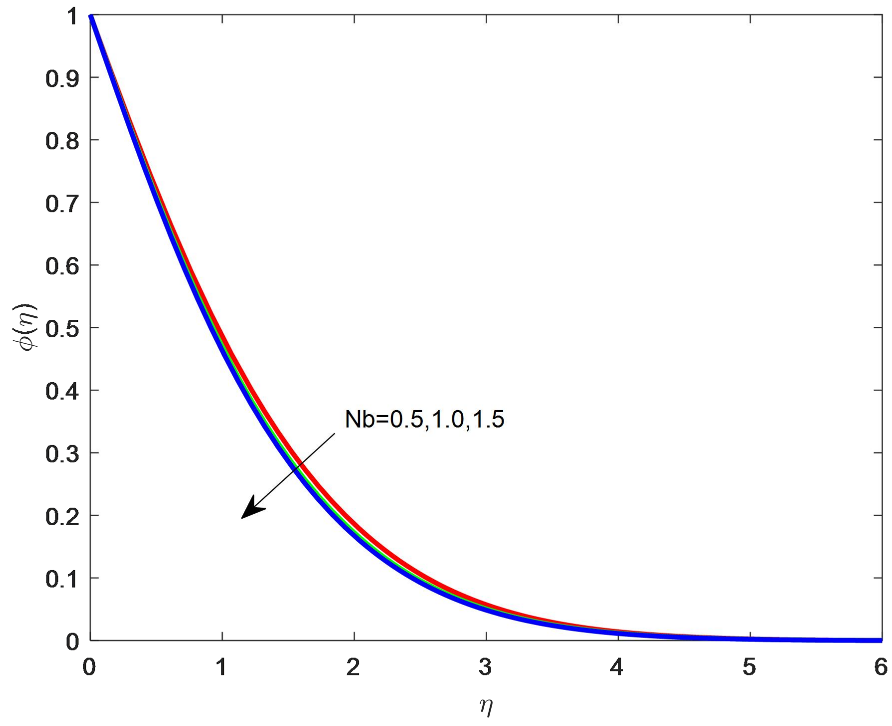

Fig 6 : Impression of Brownian motion on concnetration.

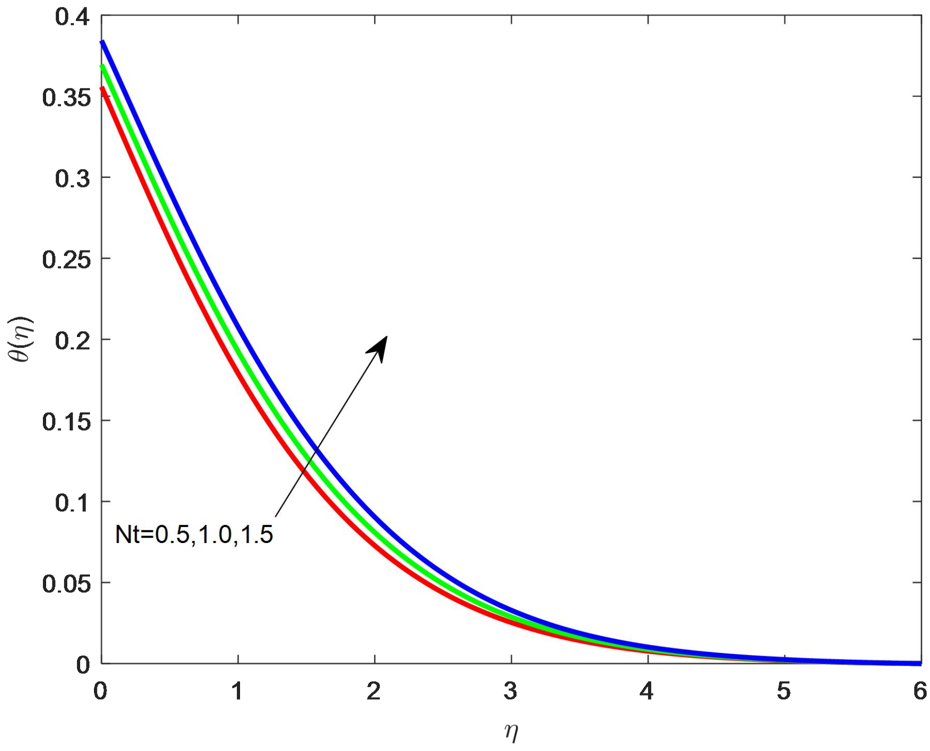

Fig 7: Impression of thermophoresis on temperature.

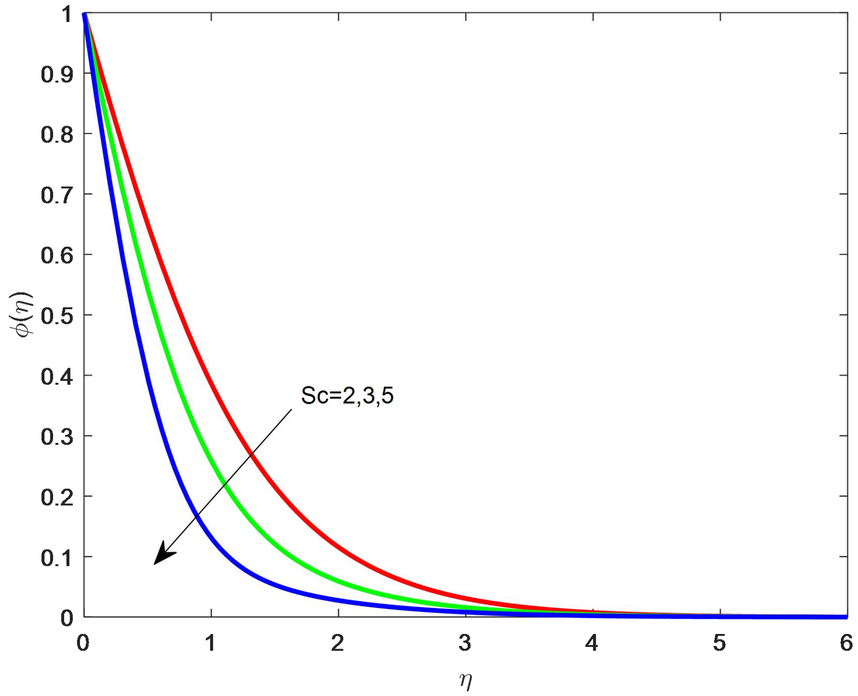

Fig 8: Significance of Schmidt number on temperature.

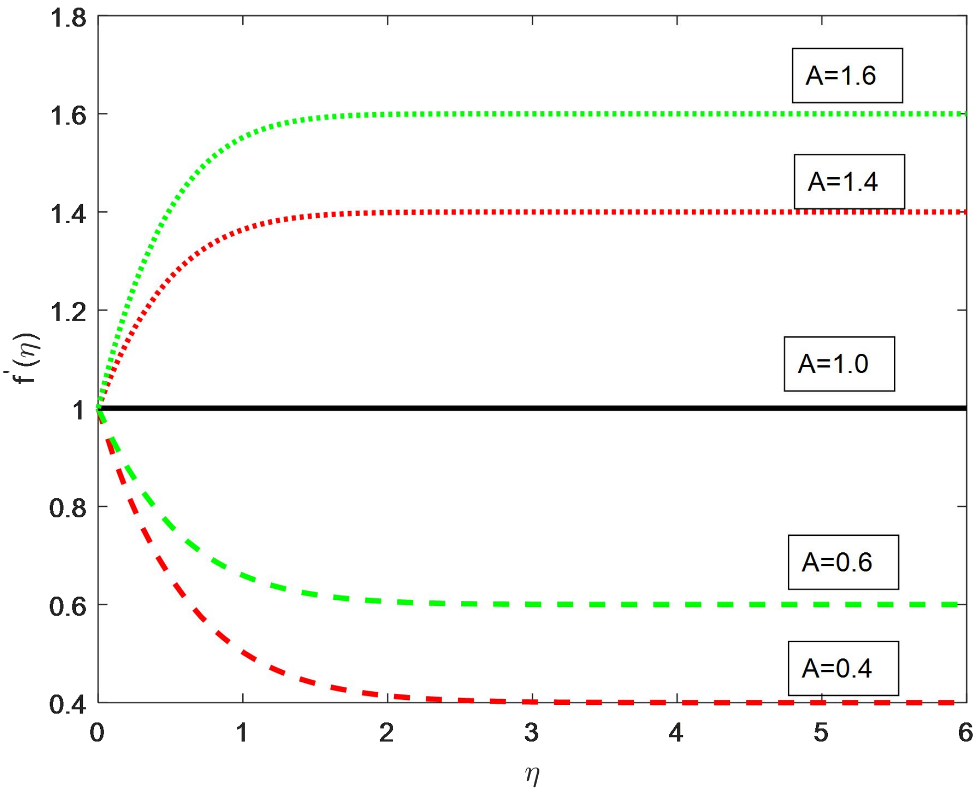

Fig 9: Significance of stretching ratio on velocity.

References :

1. Sakiadis BC. Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow. AIChE J. 1961;7(1):26–8.

2. Sakiadis BC. Boundary-layer behavior on continuous solid surfaces: II. The boundary layer on a continuous flat surface. AIChE J. 1961;7(2):221–5.

3. Sakiadis BC. Boundary-layer behavior on continuous solid surfaces: III. The boundary layer on a continuous cylindrical surface. AIChE J. 1961;7(3):467–72.

4. Akram S, Nadeem S, Hanif M. Numerical and analytical treatment on peristaltic flow of Williamson fluid in the occurrence of induced magnetic field. J Magn Magn Mater. 2013;346:142–51.

5. Hayat T, Shafiq A, Alsaedi A. Hydromagnetic boundary layer flow of Williamson fluid in the presence of thermal radiation and Ohmic dissipation. Alex Eng J. 2016 Sep 1;55(3):2229–40.

6. Halim NA, Sivasankaran S, Noor NFM. Active and passive controls of the Williamson stagnation nanofluid flow over a stretching/shrinking surface. Neural Comput Appl. 2017 Dec 1;28(1):1023–33.

7. Gorla RSR, Gireesha BJ. Dual solutions for stagnation-point flow and convective heat transfer of a Williamson nanofluid past a stretching/shrinking sheet. Heat Mass TransferWaerme- Stoffuebertragung. 2016 Jun 1;52(6):1153–62.

8. Konda JR, N.P. MR, Konijeti R, Dasore A. Effect of non-uniform heat source/sink on MHD boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium. Multidiscip Model Mater Struct. 2019 Jan 1;15(2):452–72.

9. Kumar TP, Uma MS. MHD Casson nanofluid flow over a stretching surface with melting heat transfer condition. Heat Transf. n/a(n/a).

10. Hayat T, Kiyani MZ, Alsaedi A, Ijaz Khan M, Ahmad I. Mixed convective three-dimensional flow of Williamson nanofluid subject to chemical reaction. Int J Heat Mass Transf. 2018;127:422–9.

11. Ogunseye HA, Salawu SO, Fatunmbi EO. A numerical study of MHD heat and mass transfer of a reactive Casson–Williamson nanofluid past a vertical moving cylinder. Partial Differ Equ Appl Math. 2021;4(September):100148.

12. Cattaneo. Sulla Conduzione del Calore.

13. Christov CI. On frame indifferent formulation of the Maxwell–Cattaneo model of finite-speed heat conduction. Mech Res Commun. 2009;36(4):481–6.

14. Abbasi FM, Shehzad SA. Heat transfer analysis for three-dimensional flow of Maxwell fluid with temperature dependent thermal conductivity: Application of Cattaneo-Christov heat flux model. J Mol Liq. 2016;220:848–54.

15. Anantha Kumar K, Ramana Reddy J V., Sugunamma V, Sandeep N. Magnetohydrodynamic Cattaneo-Christov flow past a cone and a wedge with variable heat source/sink. Alex Eng J. 2018;57(1):435–43.

16. Sandeep N, Sulochana C. Momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids past a stretching surface with non-uniform heat source/sink. Ain Shams Eng J. 2018 Dec 1;9(4):517–24.

17. Nazir U, Saleem S, Nawaz M, Sadiq MA, Alderremy AA. Study of transport phenomenon in Carreau fluid using Cattaneo–Christov heat flux model with temperature dependent diffusion coefficients. Phys Stat Mech Its Appl. 2020;554:123921.

18. Raju CSK, Kiran Kumar RVMSS, Varma SVK, Madaki AG, Durga Prasad P. Transpiration Effects on MHD Flow Over a Stretched Cylinder with Cattaneo–Christov Heat Flux with Suction or Injection. Arab J Sci Eng. 2018;43(5):2273–80.

19. Hayat T, Khan MI, Farooq M, Alsaedi A, Waqas M, Yasmeen T. Impact of Cattaneo-Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface. Int J Heat Mass Transf. 2016;99:702–10.

20. Rauf A, Shehzad SA, Abbas Z, Hayat T. Unsteady three-dimensional MHD flow of the micropolar fluid over an oscillatory disk with Cattaneo-Christov double diffusion. Appl Math Mech Engl Ed. 2019;40(10):1471–86.

21. Loganathan K, Sivasankaran S, Bhuvaneswari M, Rajan S. Second-order slip, cross-diffusion and chemical reaction effects on magneto-convection of Oldroyd-B liquid using Cattaneo–Christov heat flux with convective heating. J Therm Anal Calorim. 2019;136(1):401–9.

22. Ramadevi B, Kumar KA, Sugunamma V, Sandeep N. Influence of non-uniform heat source / sink on the three-dimensional magnetohydrodynamic Carreau fluid flow past a stretching surface with modified Fourier's law. Pramana - J Phys. 2019;93(6).

23. Ahmad S, Nadeem S. Flow analysis by Cattaneo–Christov heat flux in the presence of Thomson and Troian slip condition. Appl Nanosci Switz. 2020;10(12):4673–87.

24. Khan MI, Alzahrani F. Transportation of heat through Cattaneo-Christov heat flux model in non-Newtonian fluid subject to internal resistance of particles. Appl Math Mech Engl Ed. 2020;41(8):1157–66.